

Killing spinors on supersymmetric P-branes

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ABSTRACT: A class of p -brane solutions for supersymmetric gravity theories with negative cosmological constant are proposed and analyzed. The solutions are purely bosonic and contain a worldsheet and a transverse section. The classification relies on the number of intrinsic Killing spinors on the worldsheet and the transverse section. A explicit discussion of the classification is performed for the four dimensional worldsheet case.

KEYWORDS: Black Holes, Black Holes in String Theory.

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1. Introduction

The presence of a negative cosmological constant has some very interesting consequences. For instance it extends the usually called topological censorship theorem [1] allowing black holes with topologically non-trivial transverse sections [2]. Analogously, spaces which can be interpreted as extended objects, namely p -branes, are also allowed. These spaces are interesting since they may be useful as generalizations of the geometries necessary for Kaluza Klein and Randall Sundrum schemes, thus they should have a role in dimensional reduction in the presence of negative cosmological constant, $\Lambda < 0$.

In the Kaluza Klein scheme, see appendix A.1 for a review, one considers a vielbein of the form

$$\tilde{e}^i = e^i(x) \text{ and } \tilde{e}^m = \phi_l^m(x)(A^l(x) + \theta^l).$$

The above geometry is in fact a fiber bundle where, roughly speaking, \hat{e}^i describes the base and θ^l the fiber. Indeed θ^i stands for a Maurer Cartan basis on the extra dimensions. A^l stands for the gauge fields and ϕ_l^m for $(d-4)^2$ scalar fields. Here $i, j = 1 \dots p$ and $m, n = p+1 \dots p+q+1$, where p and q are the dimension of the base and the transverse section respectively. The $\{x\}$ stands for generic coordinate system on the observed p dimensions.

One can easily realize that the description of a fiber bundle with a non vanishing cosmological constant, $\Lambda \neq 0$, should differ from the $\Lambda = 0$ case. In fact, even the spaces to be casted as backgrounds differ. For $\Lambda = 0$ a natural background is a flat space, or a least Ricci flat one, which implies that the scalar fields, $\phi_l^m(x)$, be constant and the vanishing of every gauge field, A^l . Conversely, for $\Lambda \neq 0$ one should expect that the background be constant curvature manifolds, thus the extra higher dimensions need *warp* factors.

Aside of the Kaluza Klein construction arose the Randall Sundrum construction. In this case the entire space has a negative cosmological constant and unlike Kaluza Klein approach here the observed universe is located for a particular value of the radial coordinate r . The spaces considered can be described by $e^i = e^{-\frac{r}{2l}} \hat{e}^i(x)$ and $e^5 = dr$, where \hat{e}^i stands for the observed four dimensions.

In order to incorporate Kaluza Klein and Randall Sundrum schemes into a single framework with a negative cosmological constant one can consider

$$e^i = e^{-\frac{r}{2l}} \hat{e}^i(x), \quad e^5 = dr \quad \text{and} \quad e^m = e^{-\frac{r}{2l}} \phi_l^m(x) (A^l(x) + \theta^l). \quad (1.1)$$

The dimension of the space above is given by $d = p + q + 1$. It is direct to prove that this spaces in fact describes a fiber bundle on a worldsheet as the effective action reads

$$\int (R^{(d)} + \Lambda) \sqrt{g^{(d)}} d^d x \equiv \int_{\mathcal{M}_4 \times \mathbb{R}} \left(\left(R + \frac{1}{4} g_{ij} F^i_{\mu\nu} F^j{}^{\mu\nu} + \Lambda + \dots \right) (\det \phi_j^m) \right) e^{-(d-1)\frac{r}{2l}} \sqrt{g} d^4 x dr,$$

where $g_{ij}(x) = \phi_i^m(x) \phi_j^m(x) \delta_{mn}$.

Because θ^i is independent of x , the analysis of the ground state permits to determine the geometry of fiber. In the case above it is direct to check, as shown in the next sections, that the ground state of (1.1) is given by $A^l = 0$ and the spaces described by \hat{e}^a and θ^l should be at least Ricci flat. The fundamental point is that the fiber in (1.1) must be a Ricci flat manifold. Fortunately any Calabi Yau manifold is Ricci flat, however not every Calabi Yau manifold defines a ground state.

In order to generalize the geometry above to include non Ricci flat fiber one can argue that a space of the form

$$e^i = B(r) \hat{e}^i(x), \quad e^5 = C(r) dr \quad \text{and} \quad e^m = \phi_l^m(x, r) (A^l(x) + \theta^l), \quad (1.2)$$

can account for a fiber bundle with a non Ricci flat fiber. In this case the simplest candidate to be a ground state of this geometry is given by the line element

$$ds^2 = B(r)^2 (\hat{e}^i \hat{e}^j \eta_{ij}) + C(r)^2 dr^2 + A(r)^2 (\tilde{e}^m \tilde{e}^n \eta_{mn}). \quad (1.3)$$

Here \hat{e}^i and \tilde{e}^m stands for intrinsic vielbeinen on the world sheet and transverse sections respectively. Although the final idea is to consider the worldsheet the observed four dimensional world and the directions in the transverse section as the fiber still one can discuss a geometry as (1.3) in a general ground, therefore for now the worldsheet will be considered a p -dimensional manifold.

It must be noted that the solution above (1.3) is meaningful only for dimensions $d \geq 5$. In fact one should consider that the worldsheet and the transverse section at least two dimensional manifolds in order to have a non trivial case. For $d = 5$ the model is suitable for a $U(1)$ fiber.

Before to proceed, a final comment on the spaces described above is worth to be made to reinforce the aim to analyze them. Since the de Sitter group has no supersymmetric extension one could have problems to reconcile supersymmetry with the currently observed positive cosmological constant. The spaces above solve this problem nicely. The space above allows to consider a positive cosmological constant worldsheet, (\hat{e}^i) , in a supersymmetric context because it is immersed in higher dimensional negative cosmological constant space. This is actually connected with that any de Sitter space can be considered a subgroup of higher dimensional anti de Sitter group, *e.g.*, $SO(d, 1) \subset SO(d + 1, 2)$.

Killing Equation

The definition of a genuine background can be conceptually difficult. To address this problem here these spaces will be studied as solutions of a generic supergravity theory (see *e.g.* [5]). In this context a bosonic configuration, a space in this case, can be casted as a ground state if it is invariant under supersymmetry transformations. It is also a candidate to be a BPS state.

Considering a purely gravitational configuration, the arguments above reduce to determine the spaces where the equation

$$\delta\psi = \nabla\epsilon := (d + A)\epsilon = 0, \quad (1.4)$$

where A is a connection for either Poincaré or the anti de Sitter groups, can be solved. In principle one can solve this equation also for a connection for de Sitter group, however since this lacks of a supersymmetric extension this case is not usually considered. In this work only the anti de Sitter group will be considered.

It is worth to mention that for some related subjects, as a proof of the positivity of the energy [6], the existence of a supersymmetric extension is sufficient but not necessary.

In [7] this approach to identify ground states proved to be successful for spaces with topologically non-trivial transverse sections mentioned above [2]. In this work an extension of this idea will be used to classify the spaces that can represent p -brane ground states.

The connection of the anti de Sitter group is given by

$$A = \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{l}e^aJ_a, \quad (1.5)$$

where ω^{ab} is a Lorentz connection, (J_a, J_{ab}) are the generators of AdS group. $a, b = 1 \dots d$, with d the dimension of the space. l is called the AdS radius and is related to the negative cosmological constant by $\Lambda = -(d - 1)(d - 2)/(2l^2)$.

The curvature $F = dA + A \wedge A$ reads

$$F = \frac{1}{2}\bar{R}^{ab}J_{ab} + \frac{1}{l}T^aJ_a \text{ with } \bar{R}^{ab} = \left(R^{ab} + \frac{1}{l^2}e^a \wedge e^b\right), \quad (1.6)$$

where $R^{ab} = d\omega^{ab} + \omega_c^a \wedge \omega^{cb}$ is the curvature two-form and $T^a = de^a + \omega_b^a \wedge e^b$ is the torsion two form. Although in principle one could expect that spaces with non vanishing torsion could admits Killing spinors, none has been found.

Spaces

The classification of the spaces where Eq.(1.4) has solution arises from the fact that spinors may transform nontrivially under parallel transport along a closed loop. Indeed the maximal number of supersymmetries of a Euclidean manifold was shown to be determined by its holonomy group [8], which are classified by Berger's theorem. This later has been extended to semi-Riemannian manifolds, and among them Lorentzian manifolds, in [9]. The mathematical construction which allowed the classification of a simply connected, complete and irreducible Einstein manifold X of positive scalar curvature [11] (see also [12]) used the conifold mapping between X and the cone over X , which is a Ricci flat manifold. The results in [7, 10] and in the next sections can be understood as generalizations of that construction. Here it will be established a correspondence between the Killing spinors of the whole space and those of the intrinsic geometries of two sub manifolds which foliate it, a world sheet and a transverse section.

It is worth to mention that the classification of complete, connected, irreducible Riemannian (Euclidean) manifolds where Killing spinors exist is well known in the Mathematical literature [11, 13, 14, 15, 16]. This was distilled in [7] to classify the black hole ground state geometries with topologically non-trivial transverse sections. Similarly the classification for Lorentzian spaces is also known [9]. Precisely this classification will be used in this work.

Non simply connected manifolds can be obtained from simply connected ones by making identification along the orbits of the symmetries -without fixed points- of the manifold. This in general introduces noncontractible loops which may further reduce the number of Killing spinors, and thus the number of supersymmetries. Obviously this must be studied case by case.

2. An extended object and a ground state

The natural vielbein for (1.3) is given by

$$e^i = B(r)\hat{e}^i, \quad e^r = C(r)dr, \quad e^n = A(r)\tilde{e}^n. \quad (2.1)$$

By restricting to torsion free spaces the spin connection, ω^{ab} , is given by

$$\begin{aligned} \omega^{ij} &= \hat{\omega}^{ij}, \quad \omega^{mn} = \tilde{\omega}^{mn}, \\ \omega^{ir} &= \frac{B(r)'}{C(r)B(r)}e^i, \quad \omega^{mr} = \frac{A(r)'}{C(r)A(r)}e^m, \end{aligned}$$

where $\hat{\omega}^{ij}$ and $\tilde{\omega}^{mn}$ are the intrinsic Levi Civita spin connections of the world sheet and transverse section.

The Riemann curvature reads

$$\begin{aligned}
R^{ij} &= \hat{R}^{ij} - \left(\frac{\ln(B(r))'}{C(r)} \right)^2 e^i \wedge e^j \\
R^{ir} &= -\frac{1}{B(r)C(r)} \left(\frac{B(r)'}{C(r)} \right)' e^i \wedge e^r \\
R^{im} &= -\frac{\ln(B(r))' \ln(A(r))'}{C(r)^2} e^i \wedge e^m \\
R^{rm} &= -\frac{1}{A(r)C(r)} \left(\frac{A(r)'}{C(r)} \right)' e^r \wedge e^m \\
R^{mn} &= \tilde{R}^{mn} - \left(\frac{\ln(A(r))'}{C(r)} \right)^2 e^m \wedge e^n,
\end{aligned} \tag{2.2}$$

where $\hat{R}^{ij} = \hat{R}^{ij}(\hat{\omega}^{ij})$ and $\tilde{R}^{mn} = \tilde{R}^{mn}(\tilde{\omega}^{mn})$ are the intrinsical two forms of curvature of the world sheet and transverse section respectively.

From now on the language of different forms will be understood, thus the \wedge product will be omitted.

3. A constant curvature extended object

The integrability condition of the Killing spinor equation (1.4) reads

$$\nabla \nabla \epsilon = F \epsilon = 0. \tag{3.1}$$

This is trivially satisfied by the vanishing of T^a and $R^{ab} + l^{-2} e^a e^b$, therefore constant curvature manifolds are natural candidates to ground states. However in higher dimensions ($d \geq 4$) the most general solution is not necessarily a constant curvature manifold.

Non trivial negative constant curvature solutions [20] can be constructed as identifications of the form AdS/Γ where Γ is subgroup of AdS , see for instance [21], and by excising regions. This is for instance the case of the BTZ black hole [22]. Because singularities or misbehaved regions must be forbidden in the case of a ground state Γ in that case must be a subgroup of AdS without fixed points [23]. Furthermore this kind of spaces are solutions of any Lovelock gravity with a single negative cosmological constant.

From Eq.(2.2) one can determine that in order to \bar{R}^{ab} vanish then either the world sheet and the transverse section must be constant curvature manifolds. If β is the curvature of the world sheet and α the curvature of the transverse section then a generic solution reads

$$A(r)^2 = -\frac{\alpha}{\beta} B(r)^2 - \alpha \text{ and } C(r) = \sqrt{\frac{(B')^2}{\beta} + \frac{(A')^2}{\alpha}}, \tag{3.2}$$

with $B(r)$ arbitrary. The arbitrariness of $B(r)$ is consequence of that $C(r)$ could be transformed into any function by a redefinition of the radial coordinate r .

For simplicity, without loss of generality, and trying to make contact with previous known solutions one could take $\beta = -\alpha = \pm 1$ and restrict to solutions of the form

$$B(r)^2 = l^2 \left[\frac{r^2 + C_1}{(C_1 - C_2)} \right] \Rightarrow A(r)^2 = l^2 \left[\frac{r^2 + C_2}{(C_1 - C_2)} \right], \quad (3.3)$$

where C_1, C_2 are arbitrary constant. The solution with $\beta = \alpha = 0$ also exists and can be obtained from (3.3) in the limit $\sqrt{C_1} \rightarrow \sqrt{C_2}$ with a redefinition of β and α .

For an explicit example of this kind of geometries see appendix (A.2).

4. Beyond constant curvature manifolds

In the previous section was shown that constant curvature solutions, in the form of Eq.(1.3), exist. However one can readily explore a generalization of the solutions above by preserving the form of $A(r)$, $B(r)$ and $C(r)$ (see Eq.(3.3)) but leaving the world sheet or transverse section to be determined by the equations of motion. This is implicit in the model above. Indeed, these non constant curvature solutions can also be candidates to ground states. In this case the curvature is merely given by

$$\bar{R}^{ab} = \left[\begin{array}{c|c} \hat{R}^{ij} - \beta \hat{e}^i \hat{e}^j & 0 \\ \hline 0 & 0 \\ \hline 0 & \tilde{R}^{mn} + \beta \tilde{e}^m \tilde{e}^n \end{array} \right], \quad (4.1)$$

with $\beta = \pm 1, 0$. Notice that the curvature \bar{R}^{ab} is completely determined in terms of intrinsic elements of the world sheet and the transverse section.

5. Theories to be considered

As mention above the analysis of the spaces supporting Killing spinors basically is to determine the ground state of a gravitational theory. Nonetheless this only makes sense if there is a supergravity theory that supports the gravitational theory. Unfortunately, to the knowledge of these authors, only two of the Lovelock theories have well established supergravity extensions, Einstein and Chern Simons gravities. Because of that the analysis will be restricted to these cases.

The Chern Simons equation of motion ($d = 2n + 1$) reads

$$\varepsilon_b = \varepsilon_{ba_1 \dots a_{2n}} \bar{R}^{a_1 a_2} \dots \bar{R}^{a_{2n-1} a_{2n}} = 0. \quad (5.1)$$

For p and q even the equation above (5.1) reduces, considering the curvatures (4.1), to the single equation

$$(\hat{R}^{i_1 i_2} - \beta \hat{e}^{i_1} \hat{e}^{i_2}) \dots (\hat{R}^{i_{p-1} i_p} - \beta \hat{e}^{i_{p-1}} \hat{e}^{i_p}) (\tilde{R}^{l_1 l_2} + \beta \tilde{e}^{l_1} \tilde{e}^{l_2}) \dots (\tilde{R}^{l_{q-1} l_q} + \beta \tilde{e}^{l_{q-1}} \tilde{e}^{l_q}) \varepsilon_{i_1 \dots i_p l_1 \dots l_q} = 0. \quad (5.2)$$

Since the intrinsic geometries of the worldsheet and transverse section are completely independent the equation above (5.2) actually separates into the two equations

$$\begin{aligned} (\hat{R}^{i_1 i_2} - \beta \hat{e}^{i_1} \hat{e}^{i_2}) \dots (\hat{R}^{i_{p-1} i_p} - \beta \hat{e}^{i_{p-1}} \hat{e}^{i_p}) \varepsilon_{i_1 \dots i_p} &= 0 \text{ or,} \\ (\tilde{R}^{l_1 l_2} + \beta \tilde{e}^{l_1} \tilde{e}^{l_2}) \dots (\tilde{R}^{l_{q-1} l_q} + \beta \tilde{e}^{l_{q-1}} \tilde{e}^{l_q}) \varepsilon_{l_1 \dots l_q} &= 0. \end{aligned} \quad (5.3)$$

Therefore to have a solution of Eq.(5.2) is enough that either the worldsheet or the transverse section have vanishing Eq.(5.3). Unfortunately this leaves respectively either the transverse section or the world sheet undetermined. A trivial solution is therefore that either the transverse section or the worldsheet be constant curvature manifolds.

Finally, for p and q odd, the other case of Chern Simons gravity, the result is rather trivial since the equation (5.1) is identically satisfied leaving both worldsheet and transverse section undetermined. These results only confirm that Chern Simons theories are deeply complex for they have many sub-sectors with high degrees of degeneracy. This is a well known situation and has proven to be major obstacle to achieve a perturbative analysis of the Chern Simons gravity.

The Einstein-Hilbert case is straightforward and far more restrictive. In any dimension, considering (4.1), Einstein equations of motion decouple into the two set of Einstein equations

$$(\hat{R}^{i_1 i_2} - \beta \hat{e}^{i_1} \hat{e}^{i_2}) \hat{e}^{i_3} \dots \hat{e}^{i_{p-1}} \varepsilon_{i_1 \dots i_p} = 0 \text{ and } (\tilde{R}^{l_1 l_2} + \beta \tilde{e}^{l_1} \tilde{e}^{l_2}) \tilde{e}^{l_3} \dots \tilde{e}^{l_{q-1}} \varepsilon_{l_1 \dots l_q} = 0,$$

determining that the worldsheet and the transverse section satisfy Einstein equations with a cosmological constant $\pm\beta$ on their own.

6. Killing spinors and representations

Returning to the Killing spinor equation (1.4). By expressing generators in terms of d dimensional Dirac matrices as $J_{ab} = \frac{1}{2}\Gamma_{ab}$ and $J_a = \frac{1}{2}\Gamma_a$ the connection one-form A reads

$$A = \left(\frac{1}{2l} C(r) \Gamma_1 \right) + \frac{1}{2} \left(\frac{B(r)}{l} \hat{e}^i \Gamma_i + \hat{\omega}^{ij} \Gamma_i \Gamma_j \right) + \frac{1}{2} \left(\frac{A(r)}{l} \tilde{e}^m \Gamma_m + \tilde{\omega}^{mn} \Gamma_m \Gamma_n \right).$$

Formally the solution of the Killing spinor equation (1.4) reads

$$\epsilon = e^{-\Gamma_1 H(r)} \eta, \tag{6.1}$$

where

$$H(r) = \frac{1}{2l} \int C(r) dr$$

and η satisfies the equation

$$(d + \hat{A} + \tilde{A}) \eta = 0 \tag{6.2}$$

and

$$\hat{A} = \frac{1}{2} \hat{\omega}^{ij} J_{ij} + \hat{e}^i P_i, \quad \tilde{A} = \frac{1}{2} \tilde{\omega}^{mn} J_{mn} + \tilde{e}^m P_m, \tag{6.3}$$

where

$$P_i = \frac{1}{2} (P_- - \beta P_+) \Gamma_i, \quad P_m = \frac{1}{2} (P_- + \beta P_+) \Gamma_m, \tag{6.4}$$

with $P_{\pm} := \frac{1}{2}(1 \pm \Gamma_1)$.

Note that since $[P_i, P_j] = -\beta J_{ij}$, and $[P_m, P_n] = \beta J_{mn}$, therefore the sets $\{P_n, J_{mn}\}$ and the sets $\{P_n, J_{mn}\}$ form respectively *reducible* representation for $SO(d-1, 1)$, $SO(d-2, 2)$, or $ISO(d-1, 1)$ depending on whether $\beta = 1, -1$, or 0 , respectively.

The case $\beta = 0$ decouples in the following form; Let $\eta = P\eta$ which separates the Eq.(6.2) becomes

$$d\eta_+ + \frac{1}{4} (\hat{\omega}^{ij}\Gamma_{ij} + \tilde{\omega}^{mn}\Gamma_{mn}) \eta_+ = 0 \quad (6.5)$$

and

$$d\eta_- + \frac{1}{4} (\hat{\omega}^{ij}\Gamma_{ij} + \tilde{\omega}^{mn}\Gamma_{mn}) \eta_- = \frac{1}{2} (\Gamma_i \hat{e}^i + \Gamma_m \tilde{e}^m) \eta_+. \quad (6.6)$$

Flat spaces unfortunately do not have a natural scale to define, unlike constant curvature manifold where the cosmological constant defines a scale. To introduce one scale one can wrap one direction in the manifold yielding spaces of the form

$$ds^2 = d\phi^2 + d\Sigma_0, \quad (6.7)$$

where ϕ defines a circle and Σ_0 is also a $\beta = 0$ submanifold. The presence of this cycle determines that $\eta_+ = 0$ and therefore that η_- (6.6) satisfies an equation for the Lorentz group.

Nonetheless one can still consider the general case. In principle the solution of η_- can be written in terms of η_+ , which in turn satisfies an equation for the Lorentz group. The consistency condition for Eq.(6.6) gives the same information as Eq.(6.5), *i.e.*,

$$\left(\hat{R}^{ij}\Gamma_{ij} + \tilde{R}^{mn}\Gamma_{mn} \right) \eta = 0$$

Representations

Let us separate the cases according to the dimension of world sheet and transverse section. Recalling that p and q are the dimensions of the worldsheet and the transverse section respectively one can propose the following three representation according to q and p .

- **p = 2m and q = 2n.**

In this case the dimension of the space is $d = 2(m + n) + 1$, thus the dimension of the spinor η is 2^{m+n} . This allows to propose a representation where the spinor can be written as

$$\eta = \hat{\eta} \otimes \tilde{\eta}$$

where $\hat{\eta}$ and $\tilde{\eta}$ are genuine spinor on the worldsheet and the transverse section, with dimension 2^m and 2^n respectively. The representation of the Γ matrices is given by

$$\Gamma_i = \gamma_i \otimes M_\beta, \quad \Gamma_1 = \gamma \otimes \sigma \text{ and } \Gamma_m = N_\beta \otimes \sigma_m$$

where $N_1 = I_{2m} \otimes I_{2n}$, $N_{-1} = -i\gamma \otimes \sigma$ and $M_1 = i\gamma \otimes \sigma$, $M_{-1} = I_{2m} \otimes I_{2n}$. γ and σ are the proportional to γ_{2m+1} and σ_{2n+1} and satisfy $\gamma^2 = I$ and $\sigma^2 = I$ respectively.

This representation, depending on β yields the connections Eq.(6.3)

1. $\beta = -1$

$$\hat{A} = \left(\frac{1}{2} \hat{\omega}^{ij} \gamma_i \gamma_j + \frac{1}{2} \hat{e}^i \gamma_i \right) \otimes I_{2n}, \quad \tilde{A} = I_{2m} \otimes \left(\frac{1}{2} \tilde{\omega}^{mn} \sigma_m \sigma_n + \frac{i}{2} \tilde{e}^m \sigma_m \right)$$

2. $\beta = 1$

$$\hat{A} = \left(\frac{1}{2} \hat{\omega}^{ij} \gamma_i \gamma_j + \frac{i}{2} \hat{e}^i \gamma_i \right) \otimes I_{2^n} \quad \tilde{A} = I_{2^m} \otimes \left(\frac{1}{2} \tilde{\omega}^{mn} \sigma_m \sigma_n + \frac{1}{2} \tilde{e}^m \sigma_m \right)$$

3. $\beta = 0$ This case subtly different. Recalling the projection in terms of P one obtains

$$\eta_+ = \hat{\eta}_+ \otimes \tilde{\eta}_+ + \hat{\eta}_- \otimes \tilde{\eta}_-, \text{ and } \eta_- = \hat{\eta}_+ \otimes \tilde{\eta}_- + \hat{\eta}_- \otimes \tilde{\eta}_-$$

where $\gamma \hat{\eta} = \pm \hat{\eta}$ and $\sigma \tilde{\eta} = \pm \tilde{\eta}$. Restricting as mentioned before spaces with a wrapped direction of the form (6.7), which determines $\eta_+ = 0$, and the representation

$$\Gamma_i = \gamma_i \otimes I_{2^n}, \quad \Gamma_1 = \gamma \otimes \sigma \text{ and } \Gamma_m = \gamma \otimes \sigma_m$$

one can demonstrate that Eqs. (6.5, 6.6) become a Killing equation on the world sheet and transverse section provided that $\hat{\eta}_+ = \tilde{\eta}_+ = 0$ or $\hat{\eta}_- = \tilde{\eta}_- = 0$

$$\left(d + \frac{1}{4} \hat{\omega}^{ij} \gamma_{ij} \right) \hat{\eta} = 0 \quad (6.8)$$

$$\left(d + \frac{1}{4} \tilde{\omega}^{mn} \sigma_{mn} \right) \tilde{\eta} = 0 \quad (6.9)$$

This proves that the representations above indeed separates the Killing spinors equation (1.4) into worldsheet and transverse section. Therefore this proves that the problem of Killing spinors on the space has been reduced to the Killing spinor problem on the worldsheet and the transverse section.

• **$\mathbf{p = 2m + 1}$ and $\mathbf{q = 2n}$.**

In this case the dimension of the space is $d = 2(m + n) + 2$, thus the dimension of the spinor η is 2^{m+n+1} . This allows to propose a representation where the spinor can be written as

$$\eta = \hat{\eta} \otimes \tilde{\eta} \otimes \bar{\eta}$$

where $\hat{\eta}$ and $\tilde{\eta}$ are genuine spinor on the worldsheet and the transverse section, with dimension 2^m and 2^n respectively. $\bar{\eta}$ is a two dimensional constant spinor.

The representation of the Γ matrices is given by

$$\Gamma_i = \gamma_i \otimes I_{2n} \otimes \sigma_x, \quad \Gamma_1 = I_{2m} \otimes I_{2n} \otimes \sigma_z \text{ and } \Gamma_m = I_{2m} \otimes \sigma_m \otimes \sigma_y$$

$I_{2m, 2n}$ are the identity matrices in 2^m and 2^n dimensions.

In this representation, provided $\sigma_y \bar{\eta} = \pm \bar{\eta}$ when $\beta = 1$ and $\sigma_x \bar{\eta} = \pm \bar{\eta}$ when $\beta = -1$, the connection Eq.(6.3) splits as

1. $\beta = -1$

$$A\eta = \left(\frac{1}{2} \hat{\omega}^{ij} \gamma_i \gamma_j \otimes \pm \frac{1}{2} \hat{e}^i \gamma_i \right) \hat{\eta} \otimes \tilde{\eta} \otimes \bar{\eta} + \hat{\eta} \otimes \left(\frac{1}{2} \tilde{\omega}^{mn} \sigma_m \sigma_n \pm \frac{i}{2} \tilde{e}^m \sigma_m \right) \tilde{\eta} \otimes \bar{\eta}$$

2. $\beta = 1$

$$A\eta = \left(\frac{1}{2}\hat{\omega}^{ij}\gamma_i\gamma_j \mp \frac{i}{2}\hat{e}^i\gamma_i \right) \hat{\eta} \otimes \tilde{\eta} \otimes \bar{\eta} + \hat{\eta} \otimes \left(\frac{1}{2}\tilde{\omega}^{mn}\sigma_m\sigma_n \pm \frac{1}{2}\tilde{e}^m\sigma_m \right) \tilde{\eta} \otimes \bar{\eta}$$

3. $\beta = 0$ In this case using the projection in terms of P one obtains

$$\eta = \hat{\eta} \otimes \tilde{\eta} \otimes \bar{\eta}$$

where $\sigma_z\bar{\eta} = \pm\bar{\eta}$. One can demonstrate that Eqs. (6.56.6) splits on the following Killing equations on the world sheet and transverse section provided $\bar{\eta}_+ = 0$

$$\left(d + \frac{1}{4}\hat{\omega}^{ij}\gamma_{ij} \right) \hat{\eta} = 0 \quad (6.10)$$

$$\left(d + \frac{1}{4}\tilde{\omega}^{mn}\sigma_{mn} \right) \tilde{\eta} = 0 \quad (6.11)$$

As previously in this case representation also separates the Killing spinors equation (1.4) into worldsheet and transverse section. Once again the problem of Killing spinors on the space has been reduced to find Killing spinors on the worldsheet and the transverse section.

• **$p = 2m + 1$ and $q = 2n + 1$.**

This case is totally analogous to the $p = 2m + 1, q = 2n$ case analyzed before.

7. Lorentzian manifolds

In this section is summarized the classification of Lorentzian manifolds allowing Killing spinors, therefore it can be skipped for those well familiarized with the subject.

In the sections above was shown that the d dimensional Killing spinor equation reduces to effective equation on the world sheet and the transverse sections, spaces which can be either Lorentzian or Euclidean. For simplicity one can consider only effective equations. Let Σ be that manifold where the Killing equation takes the form

$$\left(d + \frac{1}{2}\omega^{AB}\gamma_A\gamma_B + \left(\frac{\sqrt{-\kappa}}{2} \right) e^A\gamma_A \right) \zeta \quad (7.1)$$

where $\kappa = 0, \pm 1$, γ^A are the corresponding Dirac matrices and ζ is a spinor. It is direct to demonstrate that $\text{sgn}(R(\Sigma)) = \text{sgn}(\kappa)$ and $R(\Sigma) = 0$ if $\kappa = 0$.

According to the value of κ the Σ spaces are given by

1. $\kappa = 0$

If the signature of Σ is (t, s) and Σ is an irreducible, simply connected and totally symmetric pseudo-Riemannian manifold then it has N Killing spinors if and only if its holonomy group H is on the table below [15].

H	t	s	N
$SU(a, b)$	$2a$	$2b$	2
$Sp(a, b)$	$4a$	$4b$	$a + b + 1$
G_2	0	7	1
$G_{2(2)}^*$	4	3	1
$G_2^{\mathbb{C}}$	7	7	2
$Spin(7)$	0	8	1
$Spin^+(4, 3)$	4	4	1
$Spin(7)^{\mathbb{C}}$	8	8	1

Note that in the classification above there are no Lorentzian manifolds. Although this seems rather restrictive actually is only due to the $\kappa = 0$ Lorentzian manifolds that admits Killing spinors are reducible, for instance Minkowski.

2. $\kappa = -1$

The analysis, in this case, is made in function of the cone C over Σ . C is defined as $-dt^2 + t^2 d\Sigma^2$ and Σ has Killing spinors with $\kappa = -1$ if and only if C has Killing spinors with $\kappa = 0$.

If C is irreducible the only possible Σ is a Lorentzian Einstein Sasaki manifold with $\dim(\Sigma)$ odd.

A most comprehensive clasification has been done in [25], in function of the Dirac current $V_\zeta^A = \bar{\zeta} \gamma^A \zeta$

Theorem *Let Σ be a Lorentzian manifold with Killing spinors with $\kappa = -1$*

1. *If Σ is not Einstein then Σ is locally conformally equivalent to a Brinkmann space with Killing spinor with $\kappa = 0$.*

2. *If $V_\zeta^A V_{A\zeta}$ is constant then*

i) *$V_\zeta^A V_{A\zeta} = 0$ and Σ is locally conformally equivalent to a Brinkmann space with Killing spinor with $\kappa = 0$.*

ii) *$V_\zeta^A V_{A\zeta} < 0$ and Σ is a Lorentzian Einstein Sasaki manifold.*

3. *If C is indecomposable and $V_\zeta^A V_{A\zeta} < 0$ then*

i) *Σ is locally conformally equivalent to a Brinkmann space with Killing spinor with $\kappa = 0$.*

ii) *Σ admits locally a warped product structure of the form*

$$dt^2 + f(t)^2 ds_{\mathcal{F}}^2$$

where $ds_{\mathcal{F}}^2$ is the line element of a Lorentzian Einstein manifold of the table below

\mathcal{F}	$f(t)$
Lorentzian Manifold with Killing spinors and $\kappa = -1$	$\cosh(t)$
Lorentzian Manifold with Killing spinors and $\kappa = 0$	e^t
Lorentzian Manifold with Killing spinors and $\kappa = 1$	$\sinh(t)$

iii) Σ is a Lorentzian Einstein Sasaki manifold (case C irreducible).

4. If $V_\zeta^A V_{A\zeta} < 0$ changes from 0 to a negative value then the region $\Omega \subset \Sigma$ where $V_\zeta^A V_{A\zeta} = 0$ is a hypersurface and $\Sigma \setminus \Omega$ admits locally a warped product structure as in 3. ii).

If the metric does not belong to the cases listed in 3. then $V_\zeta^A V_{A\zeta} < 0$ changes from 0 to a negative value or there exists a parallel 2-form which vanishes in a Riemannian subspace of C and is a pseudo-Kähler form on the complement.

3. $\kappa = -1$

Here Σ support solutions of Eq.(7.1) provided it is locally described by the line element

$$ds^2 = \sigma^2 ds_{\mathcal{F}}^2 + \varepsilon dt^2,$$

where ε is described by the table below and $ds_{\mathcal{F}}^2$ is the line element of the space \mathcal{F} also described in this table;

\mathcal{F}	σ	ε
Riemannian Manifold with Killing spinors and $\kappa = 1$	$\cosh(t)$	-1
Riemannian Manifold with Killing spinors and $\kappa = 0$	e^t	-1
Riemannian Manifold with Killing spinors and $\kappa = -1$	$\sinh(t)$	-1
Lorentzian Manifold with Killing spinors and $\kappa = 1$	$\cos(t)$	1

8. Classification of ground states with $p = 4$

Using the classification above, and that in [7], one can classify the spaces in terms of p and q . The analysis performed in this section is limited to $p = 4$ trying to make contact with the observed four dimensions such that the worldsheet could be considered as the visible world.

8.1 $\beta = 1$

In this case most general four dimensional worldsheet has locally the form of the warped product

$$ds^2 = \sigma^2 ds_{\mathcal{F}}^2 + \varepsilon dt^2.$$

The classification for $\varepsilon = -1$ is given by the following table

σ	$R_{\mathcal{F}}$	\mathcal{F}
$\cosh(t)$	1	S^3, \mathbb{RP}^3
e^t	0	$\mathbb{R}^3, S^1 \times \mathbb{R}^2, S^1 \times S^1 \times \mathbb{R}, S^1 \times S^1 \times S^1$
$\sinh(t)$	-1	$H^3, H^3/\Gamma$

where Γ is a normal subgroup of H^3 without fixed points and such that quotient manifold be non-compact.

For $\varepsilon = 1$ $\sigma = \cos(t)$ and \mathcal{F} is a three dimensional Lorentzian manifold which is classified analogously to the table above by lowering the dimension from 3 to 2. The only exception is \mathbb{RP}^2 which is non-orientable so it must be excluded from the sub classification.

8.2 $\beta = -1$

In this case one has a general family of geometries described by the stationary line element

$$ds^2 = -N(r)^2 dt^2 + \frac{1}{N(r)^2} dr^2 + r^2 d\sigma_\chi^2 \quad (8.1)$$

where $d\sigma_\chi^2$ is the line element of a two dimensional manifold with constant curvature χ listed in the table below

χ	σ_χ
1	S^2
0	$S^1 \times \mathbb{R}, \mathbb{R}^2, S^1 \times S^1$
-1	$H^2, H^2/\Gamma$

where Γ is a normal subgroup of H^2 without fixed points and such that quotient manifold be non-compact.

The rest of the spaces have non static worldsheet. One remarkable example of these spaces [9] is described by the line element

$$ds^2 = e^{2u} (dx^2 + f(x, s) ds^2 - 2ds dt) + du^2. \quad (8.2)$$

This is not an Einstein space unless $f(x, s)$ be a harmonic function on x for all s , *i.e.*, $f(x, s) = f_1(s)x + f_2(s)$.

Nonetheless in the general case the space above has a single Killing spinor given by

$$\eta = \frac{e^{-\gamma_1 \frac{u}{2}}}{f(x, s)^{\frac{1}{4}}} \eta_0 \quad (8.3)$$

where η_0 is a constant spinor that satisfies $(\gamma_0 + \gamma_3)\eta_0 = 0$ and $(1 + \gamma_1)\eta_0 = 0$.

This space above (8.2) admits a compactification along ∂_x which preserves the Killing spinor (8.3) provided $f(x, s) = f(s)$. In this case the space is an Einstein space though.

8.3 $\beta = 0$

As mentioned in the previous sections, for $\beta = 0$ there are no irreducible Lorentzian spaces having Killing spinors. This is due to the such spaces can always be constructed as a direct product of spaces of the form

$$ds^2 = ds_{\mathcal{F}}^2 - dt^2, \quad (8.4)$$

where \mathcal{F} is a Riemannian manifold with Killing spinors with $\kappa = 0$. Note that this decomposition is due to the space above is a Ricci flat manifold. Since \mathcal{F} is a three dimensional manifold with $\kappa = 0$ thus it is $\mathbb{R}^3, \mathbb{R}^2 \times S^1, \mathbb{R} \times (S^1)^2, (S^1)^3$.

8.4 Possible transverse sections

So far the possible four dimensional worldsheets have been identified. The classification of the possible transverse sections is summarized in the following table:

d	q	$\beta = 1$	$\beta = 0$	$\beta = -1$
7	2	$H^2, H^2/\Gamma$	$\mathbb{R}^2, \mathbb{R} \times S^1 \text{ or } (S^1)^2$	S^2
8	3	$H^2, H^3/\Gamma$	$\mathbb{R}^3, \mathbb{R}^2 \times S^1, \dots, (S^1)^3$	S^3, \mathbb{RP}^3
9	4	$H^4, H^4/\Gamma$	$\mathbb{R}^4, \mathbb{R}^3 \times S^1, \dots, (S^1)^4$	S^4
10	5	$H^5, H^5/\Gamma,$ $\frac{1}{z^2}(dz^2 + h_{ij}dx^i dx^j)$	$\mathbb{R}^5, \mathbb{R}^4 \times S^1, \dots, (S^1)^5$	$S^5, \mathbb{RP}^5,$ Sasaki-Einstein
11	6	$H^6, H^6/\Gamma$	$\mathbb{R}^6, \mathbb{R}^5 \times S^1, \dots, (S^1)^6$	S^6 , nearly Kähler manifold

where in the third column Γ stands for a normal subgroup of H^n without fixed points and such that H^n/Γ be non compact. h_{ij} stands for the metric of a Hyperkähler with holonomy $Sp(2)$ or a Calabi-Yau with holonomy $SU(2)$ manifold.

9. Conclusions and prospects

We have classified the families of spaces with 4-brane worldsheets that support Killing spinors and thus can be casted as genuine ground states. The classification above can be extended to any higher dimensions of the worldsheet by a careful reading of the tables in section 7.

The analysis of the four dimensional case has some interesting features. The spaces differs depending on the theory considered though.

Einstein theory In this case the equations of motion force both worldsheet and transverse section be Einstein manifolds on their own. Therefore, among the spaces permitted, four dimensional worldsheets and transverse sections, one must simply exclude the non Einstein spaces to complete the classification. This for instance forbids a worldsheet of the form (8.2).

Chern Simons theory For $d < 9$ every worldsheet in section 8 is permitted because the transverse section is a constant curvature manifold. This is particularly relevant by the presence for $\beta = -1$ of non-Einstein manifolds.

On the other hand, for $\beta = 0, 1$ any worldsheet in section 8 is permitted since every possible transverse section is either a flat or a positive curvature manifold, thus the entire space is a trivial solution of the Chern Simons equations.

For $\beta = -1$ and $d = 9, 11$, nonetheless, one must proceed with a case by case analysis to check if those spaces are solutions of Chern Simons theory. However there are fundamental examples that are worth to mention. Recalling the Chern Simons equations reduce to the multiplication of two independent equations of motion. The equation of motion for the worldsheet reads

$$\mathcal{E} = (\hat{R}^{i_1 i_2} \pm \hat{e}^{i_1} \hat{e}^{i_2})(\hat{R}^{i_3 i_4} \pm \hat{e}^{i_3} \hat{e}^{i_4})\varepsilon_{i_1 \dots i_4}.$$

Remarkably the Bohle space in Eq.(8.2) solves this equation, and thus for any transverse section in section 8.4, in $d = 9, 11$, there is a ground state.

The possible directions to continue this work are many. However, to proceed to investigate solutions over this ground states seems a natural next step. For this the presence of Calabi-Yau geometries is most relevant, in particular for the search of effective gauge theories on the worldsheet [26].

Finally it must be stressed that in the context of this work, de Sitter spaces, in particular four dimensional ones, were naturally incorporated into a supersymmetric framework. This could be most relevant to take in the current astrophysical observations and supergravity into a single unified context.

A. Appendix

A.1 Dimensional reduction

To understand the kind of space to be discussed one can review dimensional reduction. In order to make contact with the rest of this work compactification in terms of vielbeinen and spin connections, see for instance [3], will be discussed. First the presence of those higher dimensions should generate the arise of non abelian gauge theories in four dimensions. For this however the additional dimensions can not arbitrary. In particular one must consider $\mathcal{M}_d = \mathcal{M}_4 \times G_{d-4}$, being G_{d-4} a group manifold \mathbf{G} or \mathbf{G}/\mathbf{H} with \mathbf{H} a normal subgroup of \mathbf{G} [4]. For instance, the spheres $S^n = SO(n+1)/SO(n)$ and $S^{2n-1} = SU(n)/SU(n-1)$ are perfect candidates for G_{d-4} .

The ansatz for the vielbein is \tilde{e}^A , with $A = 0 \dots d$,

$$\tilde{e}^a = e^a(x) \text{ and } \tilde{e}^m = \phi_i^m(x)(A^i(x) + \theta^i) \quad (\text{A.1})$$

where θ^i is a Maurer Cartan basis on G_{d-4} . From a geometrical point of view ϕ_i^m diagonalizes the direction along the fiber, *i.e.*, it satisfies

$$\phi_i^m \phi_j^m \delta_{mn} = g_{ij}(x) \text{ and } g^{ij} \phi_i^m \phi_j^n = \delta_{mn}.$$

In four dimensions these fields actually correspond to a collection of $(d-4)^2$ scalars fields.

A spin connection compatible with the symmetries of \mathbf{G} is given by $\tilde{\omega}^{ab} = \omega^{ab}(x) + \psi^{ab}_i(x)\theta^i$, $\tilde{\omega}^{an} = \omega^{an}(x) + \hat{\omega}^{an}_j(x)\theta^j$ and $\tilde{\omega}^{mn} = \omega^{mn}(x) + \hat{\omega}^{mn}_j(x)\theta^j$. Here ψ^{ab}_i and $\hat{\omega}^{an}_j$ are scalars and ω^{ab} and $\hat{\omega}^{an}$ are one-form respectively on \mathcal{M}_4 .

The torsion free equation in d dimensions, $\tilde{T}^A = 0$, determines the effective torsion in four dimensions $T^a = -\omega^a_m \phi^m_j A^j$ where

$$\omega^{am} = -E^{a\mu} \left(\phi^m_j \frac{1}{2} F^j_{\mu\nu} dx^\nu + \partial_\mu (g_{ij}) \phi^{mj} A^i \right),$$

with $F^i = dA^i + \frac{1}{2} C^i_{jk} A^j A^k$. One can show that the contorsion is given by $K^{ab} = -g_{ij} F^{jab} A^i$. Torsion does not vanish unless A^i be pure gauge. Indeed torsion can be understood completely in terms of gauge fields.

This construction will determine the effective theory in four dimensions. For instance the Einstein Hilbert action in d dimensions is reduced to

$$I_{EH} = I_{eff} = \int_{M_d} \left(R + \frac{1}{4} g_{ij} F^i_{\mu\nu} F^{j\mu\nu} + \dots \right) \det(\phi^m_j) \sqrt{g} d^4x \varepsilon_{i_1 \dots i_{d-4}} \theta^{i_1} \dots \theta^{i_{d-4}}, \quad (\text{A.2})$$

where $g_{ij}(x) = \phi^m_i(x) \phi^m_j(x) \delta_{mn}$. Here R is the standard torsion free four dimensional Ricci scalar. The dots account for derivatives of g_{ij} .

Since this action (A.2) is independent of the coordinates on G_{d-4} one can integrate them out, thus

$$\hat{I}_{eff} = \int_{\mathcal{M}_4} \left(R + \frac{1}{4} g_{ij} F^i_{\mu\nu} F^{j\mu\nu} + \dots \right) \det(\phi^m_j) \sqrt{g} d^4x \quad (\text{A.3})$$

represents the effective Lagrangian after compactification.

A.2 An explicit solution

Using the result above Eq.(3.3) one can show that an extension of the identifications that give rise to the BTZ black hole [22] in higher dimensions leads to a space divided in three regions with

$$A(r)^2 = l^2 \left[\frac{r^2 - r_-^2}{r_+^2 - r_-^2} \right], \quad B(r)^2 = l^2 \left[\frac{r^2 - r_+^2}{r_+^2 - r_-^2} \right] \quad (\text{A.4})$$

and metric defined by

region I $r_+ < r$

$$ds_I^2 = B(r)^2 d\Sigma_1^L + ((B')^2 - (A')^2) dr^2 + A(r)^2 d\Sigma_{-1}^E, \quad (\text{A.5})$$

region II $r_- < r < r_+$

$$ds_{II}^2 = -B(r)^2 d\Sigma_{-1}^E + ((B')^2 - (A')^2) dr^2 + A(r)^2 d\Sigma_{-1}^E, \quad (\text{A.6})$$

region III $r < r_-$

$$ds_{III}^2 = -B(r)^2 d\Sigma_{-1}^E + ((B')^2 - (A')^2) dr^2 + A(r)^2 d\Sigma_1^L, \quad (\text{A.7})$$

Here $d\Sigma$ stands for the line element of a submanifold. The form of each of these (sub)manifolds is known but unnecessary for this discussion. It is enough to know that the subindexes represent the normalized curvature, and the indexes L or E stands for Lorentzian or Euclidean manifold. Therefore for $r > r_+$ the worldsheet is actually a cosmology, which for $r_- < r < r_+$ becomes an Euclidean hyperbolic space. At first sight it seems that jump occurs at $r = r_+$, however the smooth vanishing of $B^2(r)$ as $r \rightarrow r_+$ actually determines a smooth change between positive and negative curvatures.

From Eq.(3.3) one can also construct a solution without horizons which globally can be described by

$$ds_I^2 = B(r)^2 d\Sigma_{-1}^L + (-(B')^2 + (A')^2) dr^2 + A(r)^2 d\Sigma_1^E, \quad (\text{A.8})$$

where the constants in Eq.(3.3) are positive.

There is another solution, which can be casted as the *extreme* limit, $r_- \rightarrow r_+$, of the solution (A.4). As for 2+1 dimensional black hole this solution also can be obtained through an identification [22, 24]. In this solution the worldsheet and transverse sections are flat and the space is divided in two regions described by the metrics,

region outside $r_+ < r$

$$ds_{out}^2 = B(r)^2 d\Sigma_0^L + D(r)^2 dr^2 + B(r)^2 d\Sigma_0^E, \quad (\text{A.9})$$

region inside $r < r_+$

$$ds_{in}^2 = -B(r)^2 d\Sigma_0^E + D(r)^2 dr^2 - B(r)^2 d\Sigma_0^L, \quad (\text{A.10})$$

where

$$B(r)^2 = l^2 \frac{r^2 - r_+^2}{2r_+} \quad D(r)^2 = \frac{r^2}{(r^2 - r_+^2)^2}$$

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References

- [1] E. Woolgar, *Bounded area theorems for higher genus black holes*, *Class. Quant. Grav.* **16** (1999) 3005–3012, [gr-qc/9906096].
- [2] J. P. S. Lemos, *Cylindrical black hole in general relativity*, *Phys. Lett.* **B353** (1995) 46–51, [gr-qc/9404041].
- L. Vanzo, *Black holes with unusual topology*, *Phys. Rev.* **D56** (1997) 6475–6483, [gr-qc/9705004].

- D. R. Brill, J. Louko, and P. Peldan, *Thermodynamics of (3+1)-dimensional black holes with toroidal or higher genus horizons*, *Phys. Rev.* **D56** (1997) 3600–3610, [[gr-qc/9705012](#)].
- D. Birmingham, *Topological black holes in anti-de sitter space*, *Class. Quant. Grav.* **16** (1999) 1197, [[hep-th/9808032](#)].
- R.-G. Cai and K.-S. Soh, *Topological black holes in the dimensionally continued gravity*, *Phys. Rev.* **D59** (1999) 044013, [[gr-qc/9808067](#)].
- M. M. Caldarelli and D. Klemm, *Supersymmetry of anti-de sitter black holes*, *Nucl. Phys.* **B545** (1999) 434–460, [[hep-th/9808097](#)].
- R. Emparan, C. V. Johnson, and R. C. Myers, *Surface terms as counterterms in the ads/cft correspondence*, *Phys. Rev.* **D60** (1999) 104001, [[hep-th/9903238](#)].
- G. J. Galloway, K. Schleich, D. M. Witt, and E. Woolgar, *Topological censorship and higher genus black holes*, *Phys. Rev.* **D60** (1999) 104039, [[gr-qc/9902061](#)].
- R. Aros, R. Troncoso, and J. Zanelli, *Black holes with topologically nontrivial ads asymptotics*, *Phys. Rev.* **D63** (2001) 084015, [[hep-th/0011097](#)].
- [3] R. Aros, M. Romo, and N. Zamorano, *Compactification in first order gravity*, [[hep-th/0705.1162](#)].
- [4] C.-M. Y. Choquet-Brumat and M. Dillard-Bleick, *Analysis, Manifolds and Physics*. North-Holland.
- [5] E. . Salam, A. and E. . Sezgin, E., *SUPERGRAVITIES IN DIVERSE DIMENSIONS. VOL. 1, 2*. North-Holland, World Scientific, Amsterdam, Netherlands:, 1989. 1499 p.
- [6] E. Witten, *A Simple Proof Of The Positive Energy Theorem*, *Commun. Math. Phys.* **80**, 381 (1981).
- [7] R. Aros, C. Martínez, R. Troncoso, and J. Zanelli, *Supersymmetry of gravitational ground states*, *JHEP* **05** (2002) 020, [<http://arXiv.org/abs/hep-th/0204029>].
- [8] M. Wang, *Parallel spinors and parallel forms*, *Ann. Global Anal. Geom.* **7** (1989), no. 1 59–68.
- [9] C. Bohle, *Killing spinors on Lorentzian manifolds*, *Journal of Geometry and Physics* **45** (2003) 285–308.
- [10] R. Aros, C. Martínez and R. Troncoso, *Supersymmetry for extended objects with a curved worldsheet*, CECS-PHY-02/13.
- [11] C. Bär, *Real killing spinors and holonomy*, *Comm. Math. Phys.* **154** (1993) 509–521.
- [12] J. M. Figueroa-O’Farrill, *On the supersymmetries of anti de sitter vacua*, *Class. Quant. Grav.* **16** (1999) 2043–2055, [<http://arXiv.org/abs/hep-th/9902066>].
- [13] H. Baum, *Odd dimensional riemannian manifolds with imaginary killing spinors*, *Ann. Global Anal. Geom.* **7** (1989), no. 2 141–154.
- [14] H. Baum, *Complete riemannian manifolds with imaginary killing spinors*, *Ann. Global Anal. Geom.* **7** (1989), no. 3 205–226.
- [15] H. Baum and I. kath, *Parallel spinors and holonomy groups on pseudo-riemannian spin manifolds.*, *Ann. Global Anal. Geom.* **17** (1999), no. 1 1–17.
- [16] R. G. Helga Baum, Thomas Friederich and I. Kath, *Twistors and Killing Spinors on Riemannian manifolds*, vol. 124 of *Teubur text zur mathematik*. Teubur-velag, Shtugard/Leipzi, first ed., 1991.

- [17] J. T. Wheeler, *Symmetric solutions to the gauss-bonnet extended einstein equations*, *Nucl. Phys.* **B268** (1986) 737.
- [18] D. G. Boulware and S. Deser, *String generated gravity models*, *Phys. Rev. Lett.* **55** (1985) 2656.
- [19] J. Crisostomo, R. Troncoso, and J. Zanelli, *Black hole scan*, *Phys. Rev.* **D62** (2000) 084013, [[hep-th/0003271](#)].
- [20] S. Aminneborg, I. Bengtsson, S. Holst, and P. Peldan, *Making anti-de sitter black holes*, *Class. Quant. Grav.* **13** (1996) 2707–2714, [[gr-qc/9604005](#)].
- [21] M. Banados, A. Gomberoff, and C. Martinez, *Anti-de sitter space and black holes*, *Class. Quant. Grav.* **15** (1998) 3575, [[hep-th/9805087](#)].
- [22] M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, *Geometry of the (2+1) black hole*, *Phys. Rev.* **D48** (1993) 1506–1525, [[gr-qc/9302012](#)].
- [23] J. A. Wolf, *Spaces of constant curvature*, ch. 2, p. 69. Publish or perish, Wilmington, Delaware (USA), fifth ed., 1984. 412 pages.
- [24] A. R. Steif, *Supergeometry of three-dimensional black holes*, *Phys. Rev.* **D53** (1996) 5521–5526, [[hep-th/9504012](#)].
- [25] F. Leitner, *Imaginary Killing spinors in Lorentzian geometry*, *Journal of Mathematical Physics* **44** (2003) 4795–4806.
- [26] R. Aros and M. Romo, *A new form of compactification* in preparation.